CHAPTER 6

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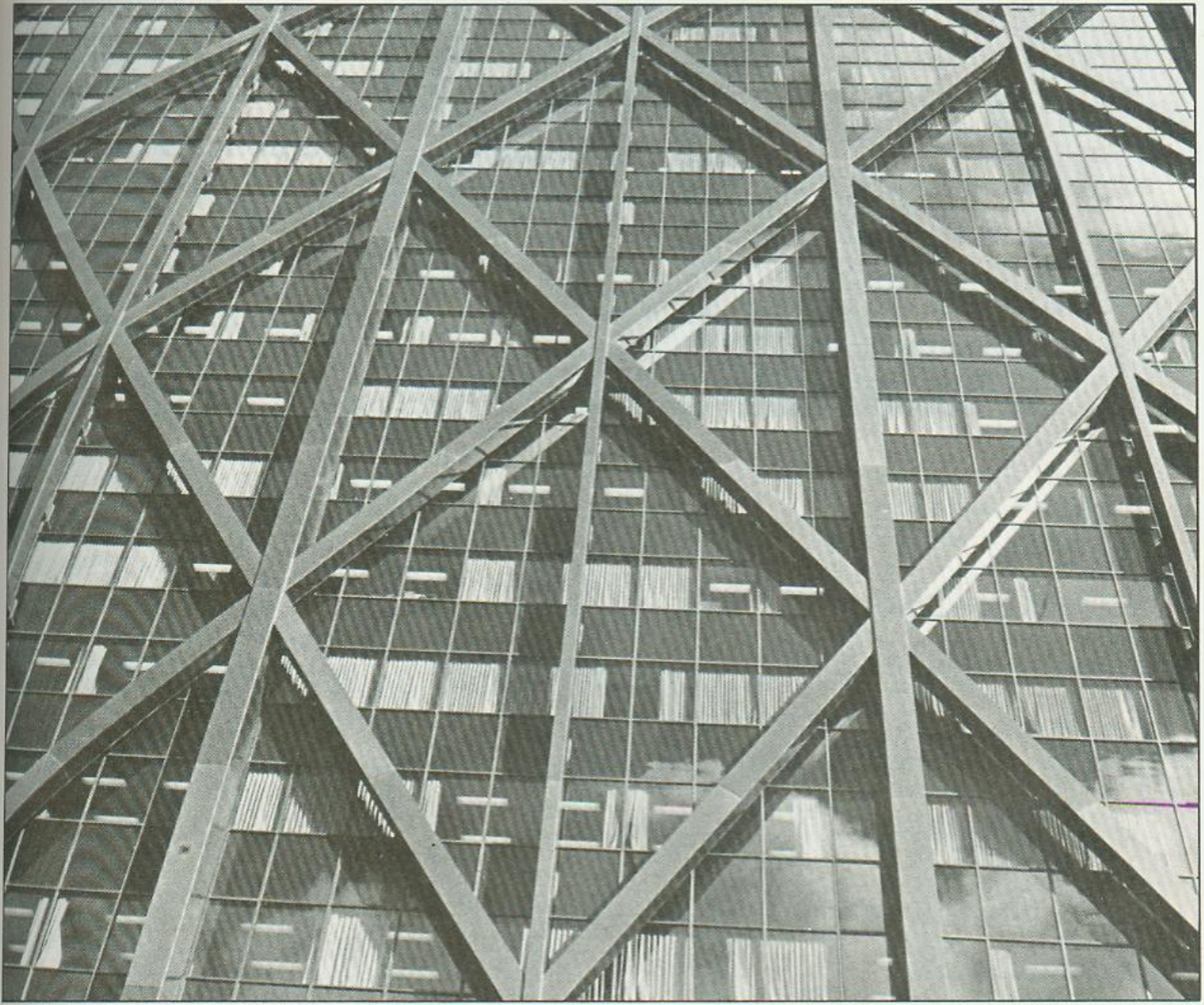
Triangles



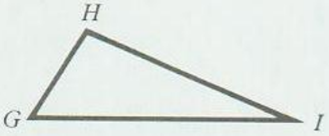
6–1 Classifying Triangles

The boat pictured here has sails that illustrate different shapes of triangles.

We can classify triangles according to the lengths of the sides or by the measure of the angles.

REVIEW: An **equilateral triangle** is a triangle with three congruent sides.

REVIEW: An **isosceles triangle** is a triangle with at least two congruent sides.

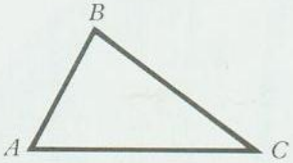


No sides are equal in length. $\triangle GHI$ is a *scalene triangle.*

Definition 6–1

A **scalene triangle** is a triangle with no congruent sides.

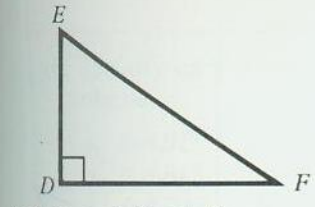
Triangles are also classified in terms of the measure of their angles.



All angles are acute. $\triangle ABC$ is an *acute triangle.*

Definition 6–2

An **acute triangle** is a triangle with three acute angles.

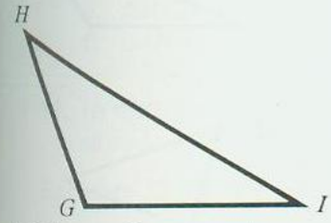


$\angle D$ is a right angle.

$\triangle DEF$ is a *right triangle*. Side $\overline{EF}$ is the *hypotenuse*. $\overline{DE}$ and $\overline{DF}$ are the *legs*.

Definition 6-3

A **right triangle** is a triangle with a right angle.

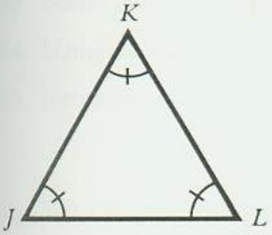


$\angle G$ is an obtuse angle.

$\triangle GHI$ is an *obtuse triangle*.

Definition 6-4

An **obtuse triangle** is a triangle with an obtuse angle.

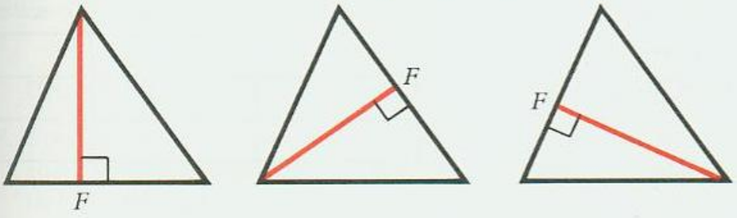


$\triangle JKL$ is an *equiangular triangle*.

Definition 6-5

An **equiangular triangle** is a triangle with three congruent angles.

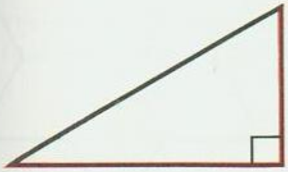
For any triangle there are three segments called *altitudes*.

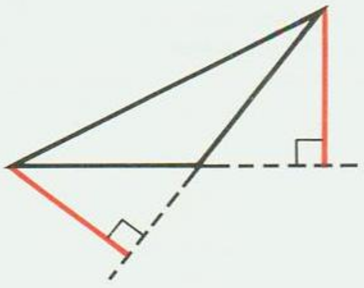


Definition 6-6

An **altitude** of a triangle is a segment from a vertex to a point F on the opposite side (perhaps extended) that is perpendicular to that opposite side.

Notice that in a right triangle two of the altitudes coincide with sides of the triangle. In an obtuse triangle two altitudes have feet on the extensions of the sides.

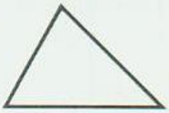


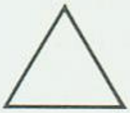


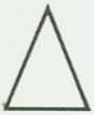
EXERCISES

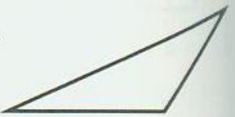
A.

1. Classify each of these triangles as scalene, isosceles, or equilateral.

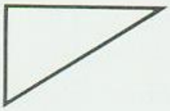
a.


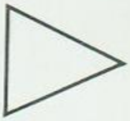
b.


c.


d.


2. Classify each of these triangles as acute, obtuse, or right.

a.


b.


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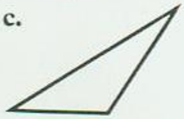
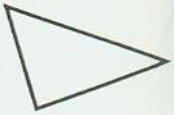
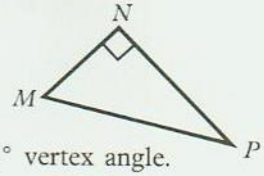


d.


3. Name these parts of right triangle *MNP*.

   a. right angle **b.** legs
   c. hypotenuse **d.** altitudes

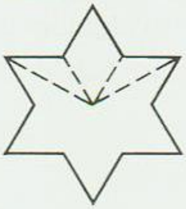


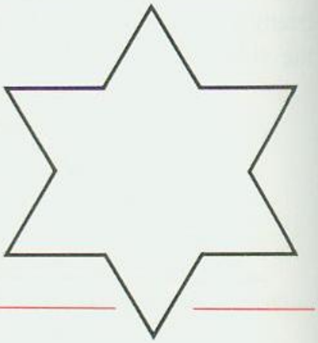
4. Construct an isosceles triangle that contains a 45° vertex angle.
5. Construct a right, scalene triangle.
6. Construct an obtuse triangle that contains a 30° angle.
7. Construct an equilateral triangle. Construct an altitude.
8. Construct an obtuse triangle. Construct all three altitudes.
9. Draw and label a large array like the one shown. In each rectangle of the array draw, if possible, a triangle satisfying both conditions.

	equilateral	isosceles	scalene
acute			
right			
obtuse			

Activity

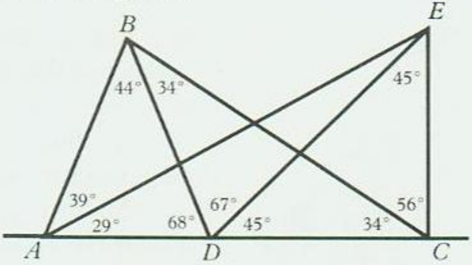
Trace three copies of the smaller star. Cut along the dotted lines and fit the pieces together to form this larger star.



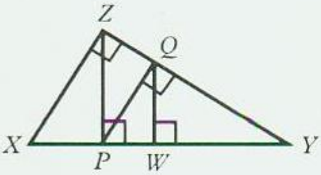


B.

10. Identify each triangle as acute, right, or obtuse.
    - a. $\triangle ABD$
    - b. $\triangle ABC$
    - c. $\triangle ADE$
    - d. $\triangle BDC$
    - e. $\triangle ACE$
    - f. $\triangle DCE$



11. In the figure on the right, identify each altitude shown and its respective triangle.

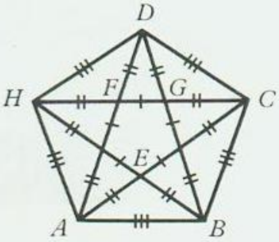


Exercises 12–14 refer to pentagon *ABCDH.*

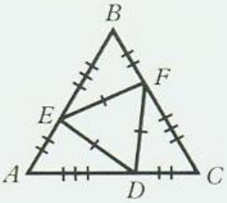
12. Name two isosceles triangles with $\overline{AB}$ the unequal side.

13. Name two isosceles triangles that have $\overline{AB}$ as one of the equal sides.

14. Name an isosceles triangle with $\overline{FG}$ as one side.



15. Name all equilateral and all scalene triangles in the figure below.

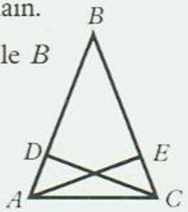


(Ex. 15)

C.

16. Why is an equilateral triangle also an isosceles triangle? Refer to the definition of "isosceles triangle" and explain.

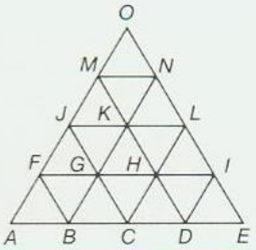
17. **Given:** Isosceles triangle ABC with vertex angle B
    $\overline{AE}$ is altitude from A to $\overline{BC}$
    $\overline{CD}$ is altitude from C to $\overline{AB}$
    $\overline{AD} \cong \overline{CE}$
    Prove: $\angle BAE \cong \angle BCD$



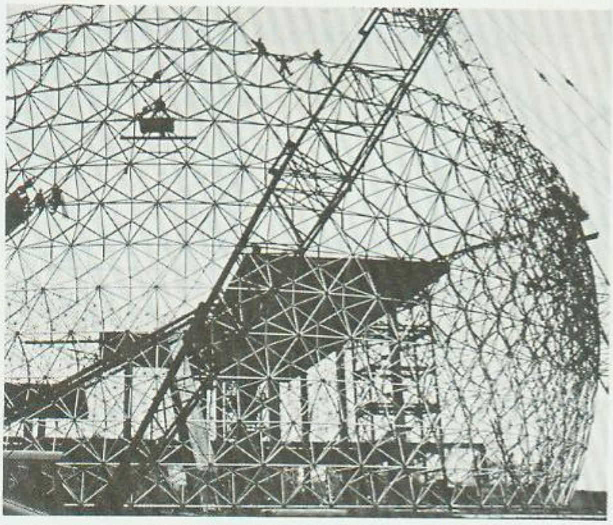
18. Four equilateral triangles can be placed to form a larger equilateral triangle. How can four obtuse triangles be placed to form a larger obtuse triangle that is the same shape as the original triangle?

PROBLEM SOLVING

There are 27 different equilateral triangles in this figure. How many of them can you name?



6–2 Isosceles Triangles

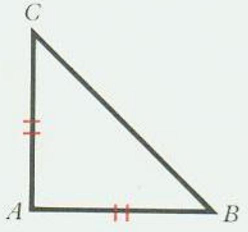


Thousands of geodesic domes have been constructed throughout the world. One of the largest was built in 1958. It is a car repair facility in Baton Rouge, Louisiana. This dome structure is 117 meters in diameter and 35 meters in height. A dome structure was also constructed for Montreal Expo in 1967. The photograph shows it under construction.

A dome structure is constructed by fitting together many triangles. Many of these triangles have sides equal in length, which means that they are either isosceles or equilateral (See Geometry in Our World, page 126).

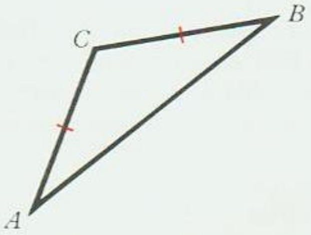
We shall study some of the important properties of these two types of triangles in this lesson.

$\overline{AB} \cong \overline{AC}$



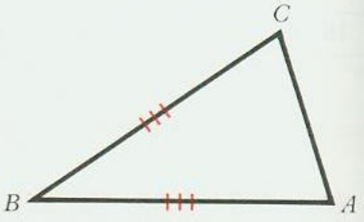
Observe that $m\angle B = m\angle C$.

$\overline{AC} \cong \overline{BC}$



Observe that $m\angle A = m\angle B$.

$\overline{AB} \cong \overline{BC}$



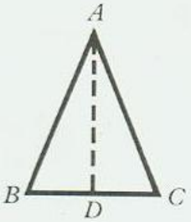
Observe that $m\angle A = m\angle C$.

Theorem 6–1 If a triangle is isosceles, then its base angles are congruent.

PROOF

Given: Let $\triangle ABC$ be isosceles with $\overline{AB} \cong \overline{AC}$.
Prove: $\angle B \cong \angle C$.

Plan: Let D be the midpoint of $\overline{BC}$. Draw $\overline{AD}$ and prove that $\triangle ABD \cong \triangle ACD$.

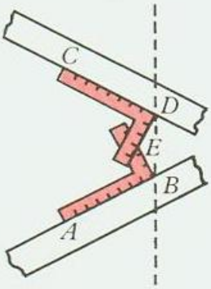


Statements	Reasons
1. $\triangle ABC$ is isosceles with $\overline{AB} \cong \overline{AC}$	1. Given
2. D is the midpoint of $\overline{BC}$.	2. Every line segment has one and only one midpoint.
3. $\triangle ABD \cong \triangle ACD$	3. A segment from the vertex angle to the midpoint of the opposite side forms a pair of congruent triangles (Theorem 4–2).
4. $\angle B \cong \angle C$	4. CPCTC

APPLICATION

Two converging timbers are to be cut along a line so that a board can be nailed to the two timbers along this line.

Two carpenter's squares are positioned so that $BE = DE$. The points B and D determine the line so that $\triangle ABD \cong \triangle CDB$. Why? The carpenter's squares are positioned so that $\triangle BDE$ is an isosceles triangle. Therefore $\angle EBD \cong \angle EDB$, and we can conclude $m\angle ABD = 90 + m\angle EBD = m\angle EDB + 90 = m\angle CDB$.



Following are two more theorems. Theorem 6–2 is a special case of Theorem 6–1.

Theorem 6–2 If a triangle is equilateral, then it is equiangular.

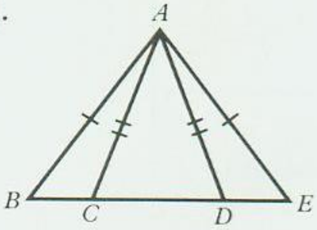
Theorem 6–3 If two angles of a triangle are congruent, then the sides opposite these angles are congruent.

EXERCISES

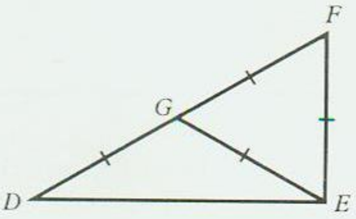
A.

In exercises 1–3 segments labeled with identical "hatch marks" are assumed to be congruent. Name all pairs of angles that are congruent by using Theorem 6–1.

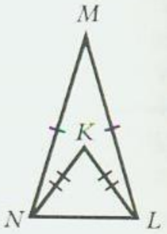
1.



2.

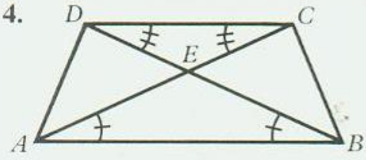


3.

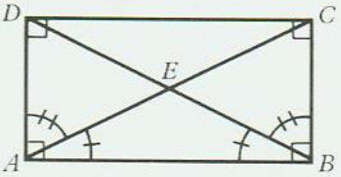


In exercises 4–6 angles labeled with identical "hatch marks" are assumed to be congruent. Name the isosceles triangles.

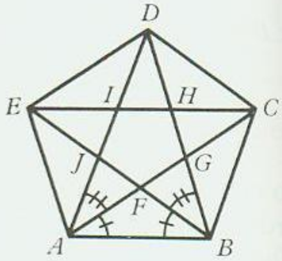
4.



5.



6.



7. $m\angle ABC = \underline{?}$

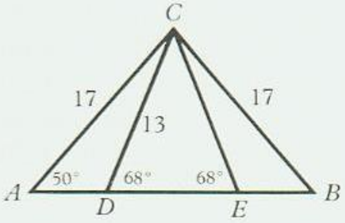
8. $CE = \underline{?}$

9. $m\angle ACD = \underline{?}$

10. $BC = \underline{?}$

11. $AB = \underline{?}$

12. $m\angle ADB = \underline{?}$



(Exs. 7, 8)

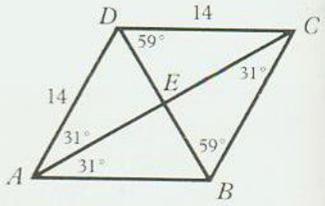
B.

13. $\triangle ABC$ is isosceles with $\overline{AB} \cong \overline{BC}$. If $AB = 4x$ and $BC = 6x - 15$, find AB and BC.

14. In $\triangle DEF$, $\overline{DE} \cong \overline{EF}$. If $DE = 4x + 15$, $EF = 2x + 45$, and $DF = 3x + 15$, find the lengths of the sides of the triangle.

15. In $\triangle XYZ$, $\overline{XY} \cong \overline{YZ}$. If $m\angle X = 4x + 60$, $m\angle Y = 2x + 30$, and $m\angle Z = 14x + 30$, find $m\angle X$, $m\angle Y$, and $m\angle Z$.

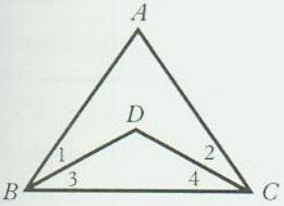
16. In $\triangle ABC$, $\angle A \cong \angle C$. If $AB = 4x + 25$, $BC = 2x + 45$, and $AC = 3x - 15$, find the lengths of the three sides.



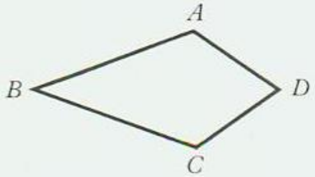
(Exs. 9–12)

For exercises 17-24, write two-column proofs.

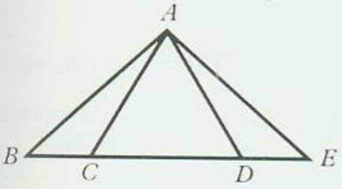
17. **Given:** $\overline{AB} \cong \overline{AC}$
$\overline{DB} \cong \overline{DC}$
Prove: $\angle 1 \cong \angle 2$



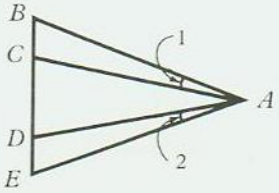
18. **Given:** $ABCD$ is a quadrilateral with $\overline{AB} \cong \overline{BC}$ and $\overline{DA} \cong \overline{DC}$.
Prove: $\angle BAD \cong \angle BCD$
(*Hint:* Add an auxiliary line.)



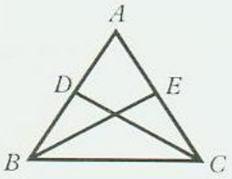
19. **Given:** $\overline{AC} \cong \overline{AD}$, $\overline{BC} \cong \overline{ED}$
Prove: $\triangle ABE$ is isosceles.



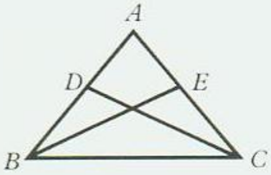
20. **Given:** $\overline{AC} \cong \overline{AD}$, $\angle 1 \cong \angle 2$
Prove: $\triangle ABD \cong \triangle AEC$



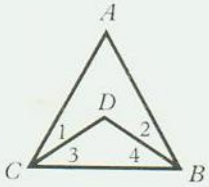
21. **Given:** $\overline{AB} \cong \overline{AC}$
$\overline{BE}$ bisects $\angle B$.
$\overline{CD}$ bisects $\angle C$.
Prove: $\overline{CD} \cong \overline{BE}$



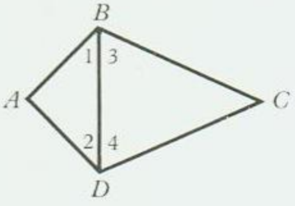
22. **Given:** $\overline{AB} \cong \overline{AC}$
$\overline{BE}$ bisects $\angle B$.
$\overline{CD}$ bisects $\angle C$.
Prove: $\overline{AD} \cong \overline{AE}$



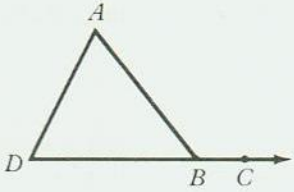
23. **Given:** $\overline{AB} \cong \overline{AC}$
$\angle 1 \cong \angle 2$
Prove: $\triangle BCD$ is isosceles.



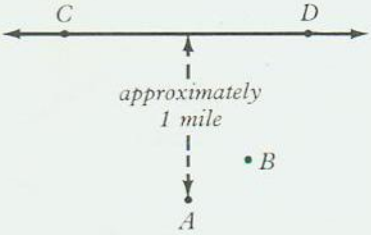
24. **Given:** $\angle ABC \cong \angle ADC$
$\angle 1 \cong \angle 2$
Prove: $\overline{AB} \cong \overline{AD}$ and $\overline{BC} \cong \overline{DC}$



25. **Given:** $\angle ABC$ and $\angle D$ are supplementary.
Prove: $\triangle ABD$ is isosceles.

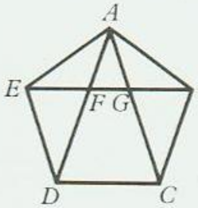


26. A survey team has been asked to locate a point on property line CD that is equal in distance from two previously located survey pins A and B. Because of the magnitude of the distances, direct measurement with a tape measure is not feasible. Points A and B are close together. Transits are located at each point. How can the team locate the desired point?



C.

In exercises 27–28, $ABCDE$ is a regular pentagon.



(Exs. 27, 28)

27. Prove that $\triangle ABG$ is an isosceles triangle.

28. Prove that $\triangle AFG$ is an isosceles triangle.

29. Prove Theorem 6–2.

30. Prove Theorem 6–3.

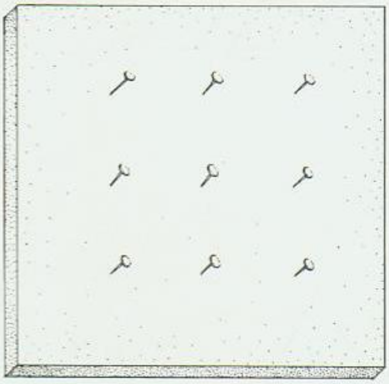
31. Prove that the bisector of the vertex angle of an isosceles triangle is the perpendicular bisector of the opposite side.

32. Prove that if $\triangle ABC \cong \triangle DEF$ and $\triangle DEF \cong \triangle GHI$, then $\triangle ABC \cong \triangle GHI$. (Transitive property of congruent triangles.)

Activity

On a geoboard with 3 nails on a side or on 3-by-3 dot paper show:

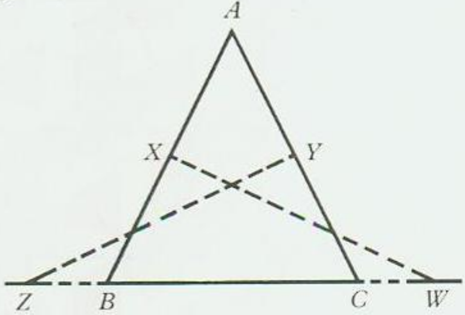
1. Segments of five different lengths.
2. Angles of ten different sizes.
3. Triangles of seven different sizes and shapes.



33. Prove that the bisectors of the base angles of an isosceles triangle are congruent.

34. Prove that the line segments joining the midpoint of the base of an isosceles triangle to the midpoints of the legs are congruent.

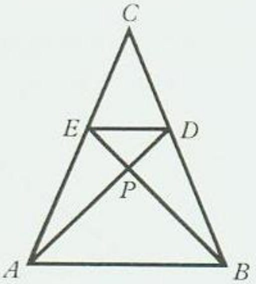
35. In triangle ABC, $\overline{AB} \cong \overline{AC}$. X is the midpoint of $\overline{AB}$ and Y is the midpoint of $\overline{AC}$. $\overleftrightarrow{XW}$ is drawn perpendicular to $\overline{AB}$; $\overleftrightarrow{YZ}$ is drawn perpendicular to $\overleftrightarrow{AC}$; W and Z are points on $\overleftrightarrow{BC}$. Prove that $\overline{XW} \cong \overline{YZ}$.



(Ex. 35)

36. Prove that an equiangular triangle is equilateral.

37. Suppose that $\triangle ABC$ is an isosceles triangle with $\overline{AC} \cong \overline{BC}$, and that $\overline{AD}$ and $\overline{BE}$ intersect at P so that $\angle PAB \cong \angle PBA$. Prove that $\triangle PDE$ is an isosceles triangle.

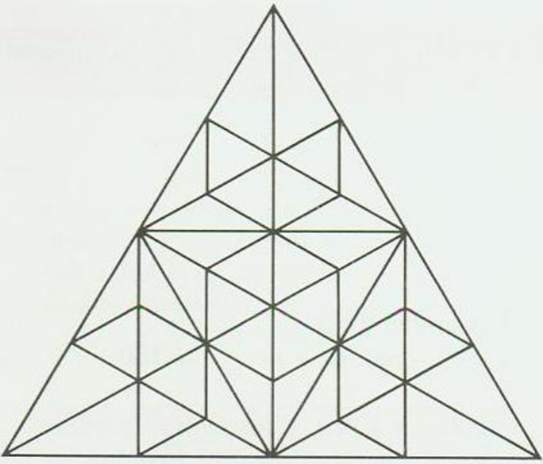


(Ex. 37)

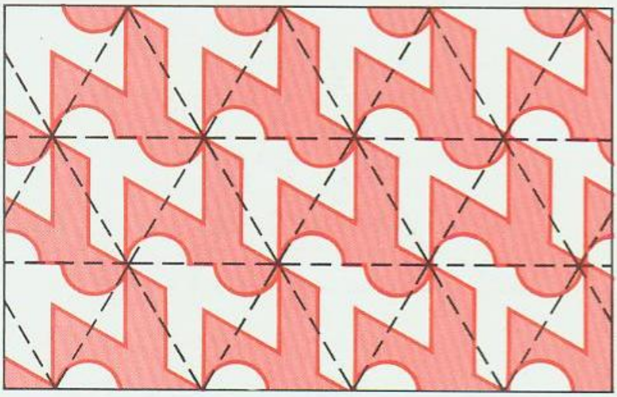
PROBLEM SOLVING

How many equilateral triangles are there?

How many kites are there? (A kite is a quadrilateral with exactly two pairs of congruent adjacent sides.)



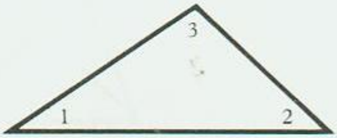
6–3 Measures of the Angles of a Triangle

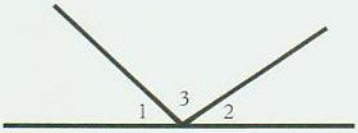


Geometric patterns and designs are interesting and important to an interior decorator. (See Geometry In Our World, p. 40.) Many such patterns, when carefully analyzed, are built around a triangular shape that is repeated.

The theorem in this section describes a property of triangles that gives them a "patternmaking" quality.

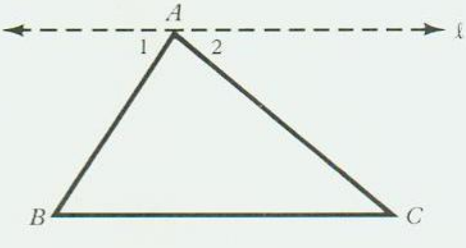
In Chapter 2 we saw that if the corners of a triangle are cut off and fitted together, the sum of the angles appears to be 180°.





We now state this as a theorem.

Theorem 6–4 The sum of the measures of the angles of a triangle is 180°.



PROOF

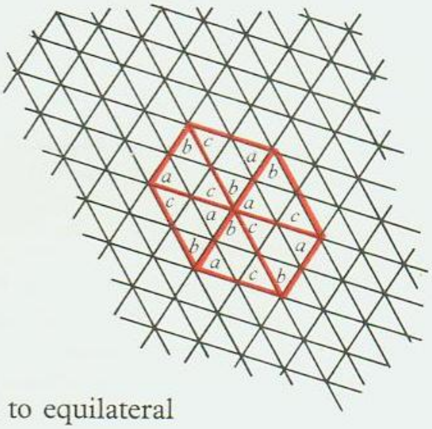
Given: A triangle ABC
Prove: $m\angle A + m\angle B + m\angle C = 180$

Plan: Construct a line ℓ through A parallel to $\overline{BC}$, and use theorems relating parallel lines and a transversal. (Line ℓ is an auxiliary line.)

Statements	Reasons
1. Let ℓ be a line through A parallel to $\overline{BC}$.	1. Construction
2. $\angle 1 \cong \angle B$, $\angle 2 \cong \angle C$	2. If two lines are parallel, then the alternate interior angles are congruent.
3. $m\angle 1 + m\angle A + m\angle 2 = 180$	3. Definition of between for rays and Linear Pair Postulate
4. $m\angle B + m\angle A + m\angle C = 180$	4. Substitution

APPLICATION

Since the sum of the measures of the angles of a triangle is 180° (in the figure $a + b + c = 180$) congruent copies of a triangle can be arranged around a point as shown in the figure and extended to cover the plane. These triangular patterns form the basis for more intricate patterns as shown at the beginning of the lesson.

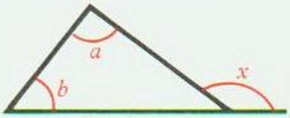


The next theorem can be proved by applying Theorem 6-4 to equilateral triangles.

Theorem 6–5 The angles of an equilateral triangle each have a measure of 60°.

The theorem stated below is useful when solving certain geometric problems. It is stated and illustrated here and will be proved as an exercise.

$x = a + b$, where x is the measure of the exterior angle and a and b are measures of remote interior angles.



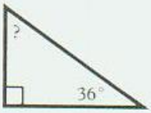
Theorem 6–6 The measure of an exterior angle of a triangle is equal to the sum of the measures of its two remote interior angles.

EXERCISES

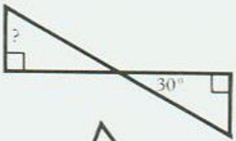
A.

Find the measures of the indicated angles in exercises 1–9. Hatch marks indicate congruent segments.

1.



2.



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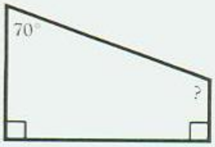

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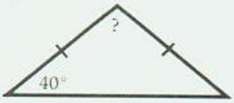

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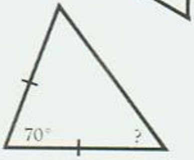

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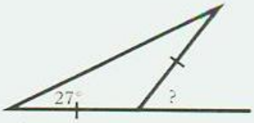

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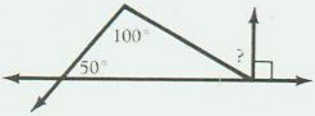

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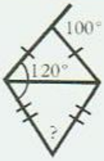
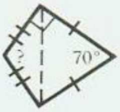


9.



B.

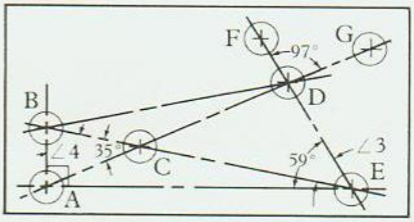
10. The angle measures of the base angles of an isosceles triangle are represented by x and the vertex angle by $2x + 30$. Find the measures of each angle.

11. The measures of the angles of a triangle are represented by $2x + 15$, $x + 20$, and $3x + 25$. Find the measures of the angles.

12. Prove that the angles of an equilateral triangle each have a measure of 60°.

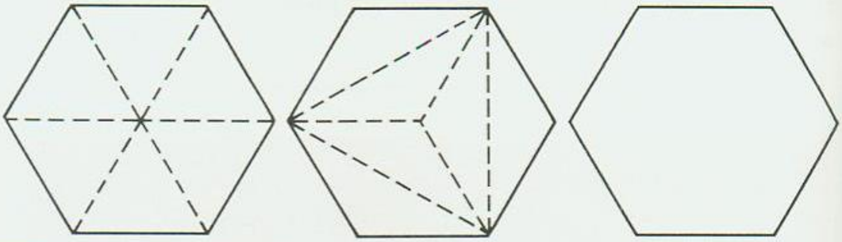
13. Prove that the base angles of an isosceles right triangle each have a measure of 45°.

14. A machinist must make a steel plate with holes drilled as shown. The machinist must first calculate the measures of $\angle 3$ and $\angle 4$. Find $m\angle 3$ and $m\angle 4$.



Activity

Trace and cut out the three small hexagons and cut along all the dotted lines. Use the 13 pieces to form a large regular hexagon.



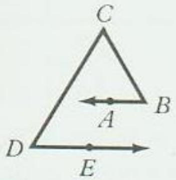
15. Prove that the base angles of an isosceles triangle are acute.

16. In $\triangle ABC$ and $\triangle DEF$, $\angle A \cong \angle D$ and $\angle B \cong \angle E$. Prove that $\angle C \cong \angle F$.

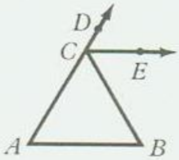
17. Prove that the acute angles of a right triangle are complementary.

C.

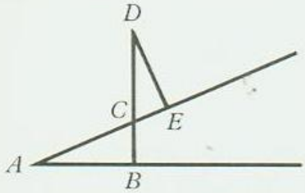
18. **Given:** $\overleftrightarrow{AB} \parallel \overleftrightarrow{DE}$
Prove: $m\angle B + m\angle C + m\angle D = 180$



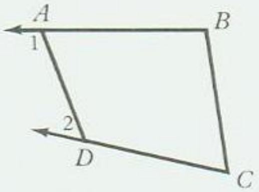
19. **Given:** $\overrightarrow{CE}$ bisects $\angle BCD$.
$\overline{AC} \cong \overline{BC}$
Prove: $\overleftrightarrow{CE} \parallel \overleftrightarrow{AB}$



20. **Given:** $\overline{DE} \perp \overline{AE}$
$\overline{DB} \perp \overline{AB}$
Prove: $m\angle CAB = m\angle CDE$



21. **Given:** $\overleftrightarrow{AB}$, $\overleftrightarrow{BC}$, $\overleftrightarrow{CD}$, and $\overleftrightarrow{AD}$ as shown
Prove: $m\angle 1 + m\angle 2 = m\angle C + m\angle B$



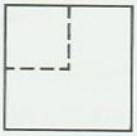
22. Prove that the sum of the angles of a quadrilateral is 360°.

23. Prove Theorem 6–6.

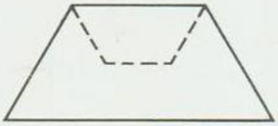
PROBLEM SOLVING

Trace each figure and finish drawing the dotted lines to divide each figure into four identical parts, each the same shape as the original figure.

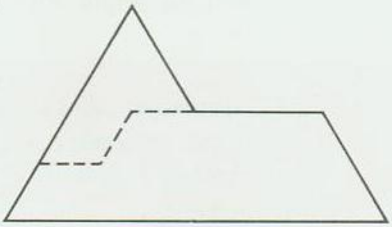
A



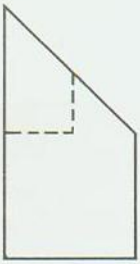
B



C



D



6–4 The AAS Congruence Theorem

On cannibal island
there lay some trees,

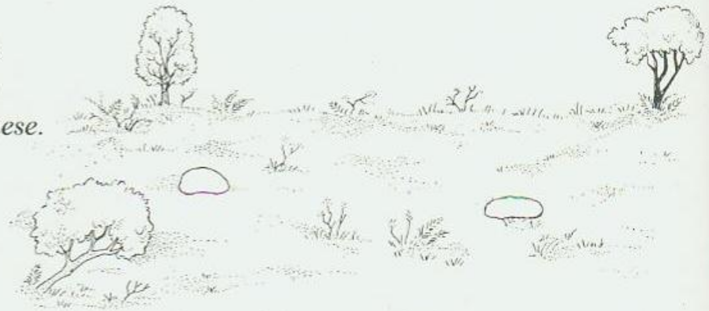
An oak, a maple, and an elm chinese.

As ye strike a path
from elm to oak

You'll find the treasure
of which I've spoke.

Just find the spot
and bet your socks

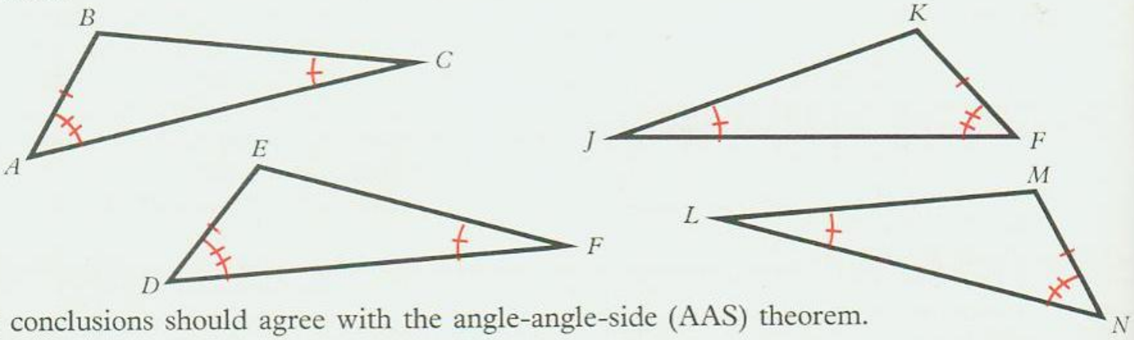
On a triangle congruent to
my triangle of rocks.



If you find a spot X where the angle is the same as the corresponding angle in the rock triangle, can you be sure that the triangles are congruent?

A pirate who was a frustrated geometer laid out three large rocks, and wrote the poem shown above.

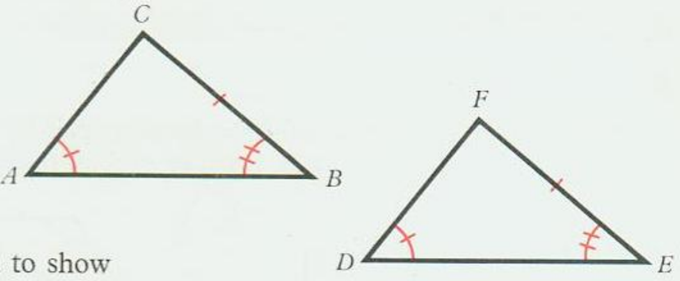
In the triangles below, two angles and a side opposite one angle are congruent. Use tracing paper to convince yourself that the triangles are congruent.



Your conclusions should agree with the angle-angle-side (AAS) theorem.

Theorem 6–7 **AAS Theorem.** If two angles and a side opposite one angle in a triangle are congruent to two angles and the corresponding side of a second triangle, then the two triangles are congruent.

PROOF



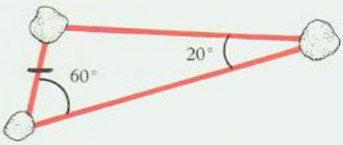
Given: $\triangle ABC$ and $\triangle DEF$ with
$\angle A \cong \angle D$
$\angle B \cong \angle E$
$\overline{BC} \cong \overline{EF}$

Prove: $\triangle ABC \cong \triangle DEF$

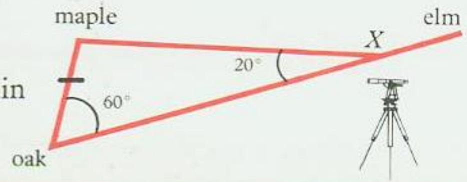
Plan: We shall use the given information to show that $\angle C \cong \angle F$ and then use the ASA postulate.

Statements	Reasons
1. $\angle A \cong \angle D$, $\angle B \cong \angle E$	1. Given
2. $m\angle A + m\angle B + m\angle C = 180$; $m\angle D + m\angle E + m\angle F = 180$	2. The sum of the measures of the three angles of a triangle is 180°.
3. $m\angle A + m\angle B + m\angle C = m\angle D + m\angle E + m\angle F$	3. Substitution
4. $m\angle C = m\angle F$	4. Subtraction of equals property
5. $\angle C \cong \angle F$	5. Definition of congruent angles
6. $\overline{BC} \cong \overline{EF}$	6. Why?
7. $\triangle ABC = \triangle DEF$	7. Why?

APPLICATION



Theorem 6–7 provides an answer to the puzzle problem on page 212. Since two angles and a side opposite one of the angles of the "tree triangle" are congruent to two angles and the side opposite one of the angles of the "rock triangle," the two triangles are congruent and X marks the spot of the treasure!



When Theorem 6–7 is applied to a right triangle we obtain the hypotenuse-angle (HA) theorem stated below.

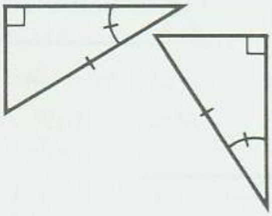
Theorem 6–8 **HA Theorem.** If the hypotenuse and an acute angle of one right triangle are congruent to a hypotenuse and an acute angle of another right triangle, then the triangles are congruent.

EXERCISES

A.

For exercises 1–6 indicate whether the given pair of triangles is congruent by AAS, HA, SAS, ASA, or none of these.

1.



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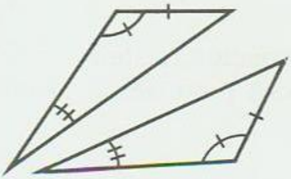

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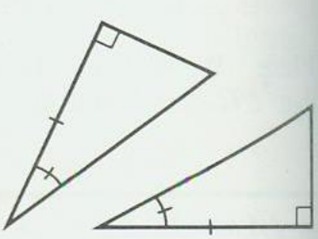

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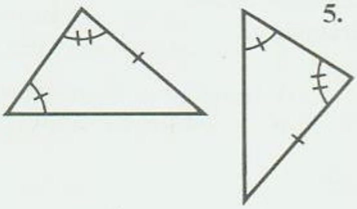

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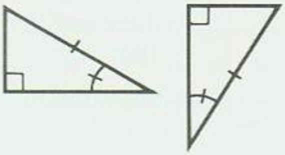

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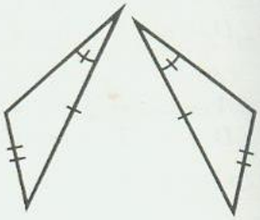
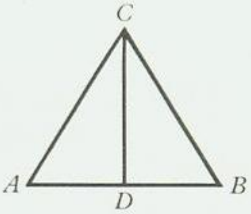


B.

Using the information given in exercises 7–8, prove that $\triangle ACD \cong \triangle BCD$.

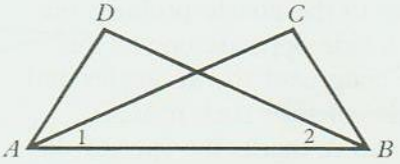


(Exs. 7, 8)

7. Given: $\overline{CD} \perp \overline{AB}$
$\angle A \cong \angle B$

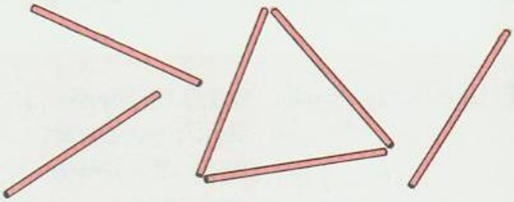
8. Given: $\overline{CD}$ bisects $\angle C$.
$\angle A \cong \angle B$

9. Given: $\angle 1 \cong \angle 2$
$\angle C \cong \angle D$
Prove: $\triangle ABC \cong \triangle BAD$

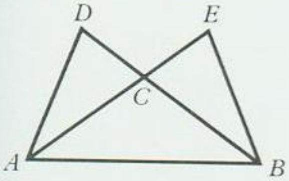


Activity

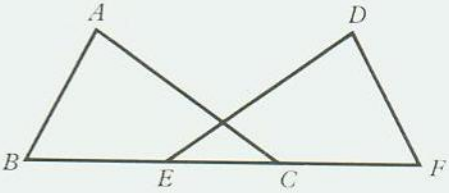
Cut straws of the following lengths:
two 10 cm long
two 17.3 cm long
one 20 cm long
one 30 cm long
How many different shaped triangles can be made using these six straws?



10. **Given:** $\triangle ABC$ is isosceles with vertex angle C.
$\angle D \cong \angle E$
Prove: $\triangle ABE \cong \triangle BAD$



11. **Given:** $\overline{BE} \cong \overline{CF}$
$\angle A \cong \angle D$
$\angle ACB \cong \angle DEF$
Prove: $\triangle ABC \cong \triangle DFE$

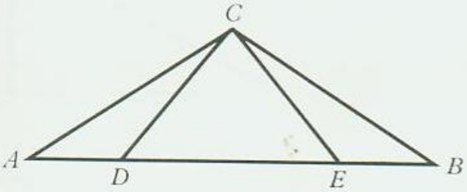


12. Write a two-column proof for Theorem 6–7.

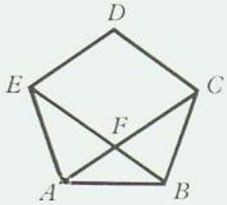
13. Write a two-column proof for Theorem 6–8.

C.

14. **Given:** $\angle A \cong \angle B$
$\angle ADC \cong \angle BEC$
Prove: $\triangle AEC \cong \triangle BDC$



15. **Given:** $ABCDE$ is a regular pentagon.
Prove: $\triangle AFE \cong \triangle BFC$

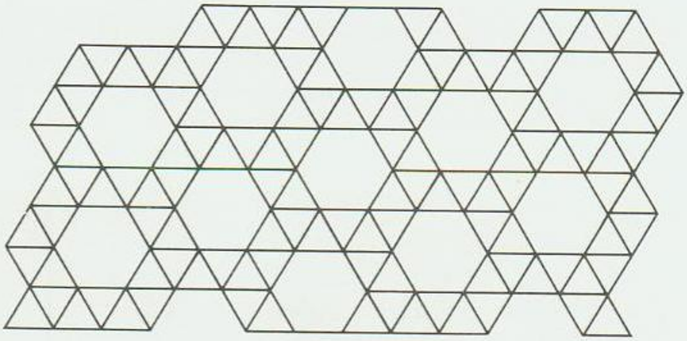


16. Prove that the segments from a point on an angle bisector perpendicular to the sides of the angle are congruent.

17. Prove that if perpendiculars are drawn from any point on the base of an isosceles triangle to the congruent sides, the angles formed on the base of the triangle are congruent.

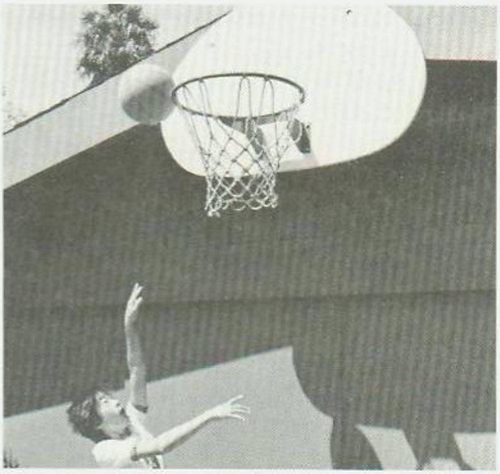
PROBLEM SOLVING

If the triangles and hexagons in this pattern were colored so that any two polygons with a common vertex had different colors, what is the smallest number of colors required to accomplish this coloring?



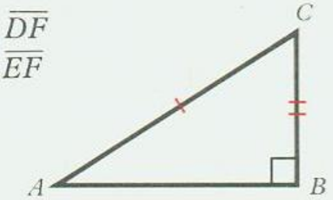
6–5 The HL Congruence Theorem

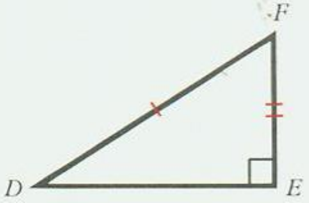
Suppose you want to install an outdoor basketball backstop to a wall. How can you be sure that the backstop will be parallel to the wall? The braces connecting the backstop to the wall are important in the answer.



Consider the following pairs of triangles.

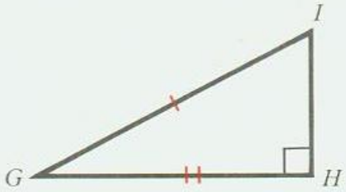
$\overline{AC} \cong \overline{DF}$
$\overline{BC} \cong \overline{EF}$

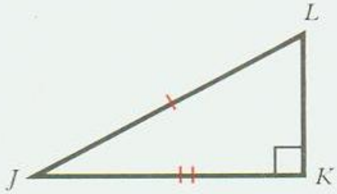




Use tracing paper to conclude:
$\triangle ABC \cong \triangle DEF$

$\overline{GI} \cong \overline{JL}$
$\overline{GH} \cong \overline{JK}$



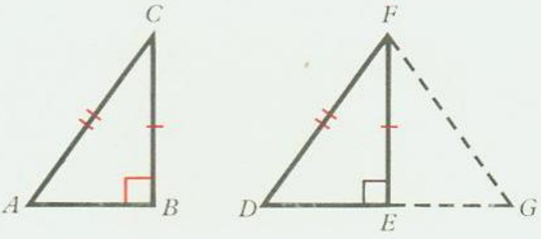


$\triangle GHI \cong \triangle JKL$

Theorem 6–9 **HL Theorem.** If the hypotenuse and a leg of one right triangle are congruent to the hypotenuse and a leg of a second right triangle, then the triangles are congruent.

PROOF

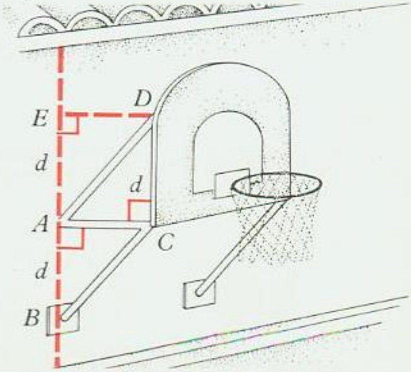
Given: $\triangle ABC$ and $\triangle DEF$ with $\angle B$ and $\angle E$ right angles. $\overline{BC} \cong \overline{EF}$, and $\overline{AC} \cong \overline{DF}$.
Prove: $\triangle ABC \cong \triangle DEF$



Statements	Reasons
1. Draw $\overrightarrow{DE}$	1. Construction
2. Choose G on $\overrightarrow{DE}$ so that $\overline{EG} \cong \overline{AB}$	2. Choice of G
3. $\angle ABC$ and $\angle DEF$ are right angles	3. Given
4. $\angle GEF$ is a right angle	4. If one angle of a linear pair is a right angle, then the other is a right angle.
5. $m\angle ABC = m\angle DEF = m\angle GEF = 90$	5. Definition of right angle
6. $\angle ABC \cong \angle DEF \cong \angle GEF$	6. Definition of congruent angles
7. $\overline{BC} \cong \overline{EF}$	7. Given
8. $\triangle ABC \cong \triangle GEF$	8. Why?
9. $\overline{AC} \cong \overline{GF}$	9. CPCTC
10. $\overline{AC} \cong \overline{DF}$	10. Given
11. $\overline{GF} \cong \overline{DF}$	11. Transitive property of segment congruence
12. $\angle FDE \cong \angle FGE$	12. If a triangle is isosceles, then its base angles are congruent
13. $\overline{EF} \cong \overline{EF}$	13. Why?
14. $\triangle DEF \cong \triangle GEF$	14. AAS Theorem
15. $\triangle ABC \cong \triangle DEF$	15. Transitive property of congruent triangles (See p. 206, Ex. 32)

APPLICATION

Consider the question asked at the beginning of the lesson. The backstop should be mounted so that $\overline{AC} \perp \overline{AB}$ and $AB = CD$. In order for the backboard to be parallel to the wall, $\overline{AC}$ and $\overline{ED}$ must be equal in length. If $\overline{AD} \cong \overline{BC}$, then $\triangle ABC \cong \triangle EAD$ by Theorem 6-9 (HL) and $\overline{AC} \cong \overline{ED}$. Notice that in order to have the congruent triangles, we had to have braces $\overline{AD}$ and $\overline{BC}$ be the same length.



Theorem 6-9 is used in the proof of this next important theorem.

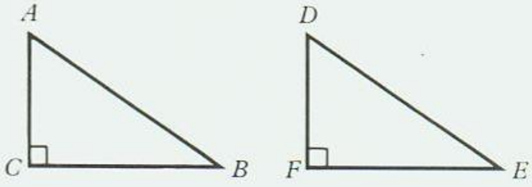
Theorem 6–10 If a point P is equidistant from a pair of points A and B, then P lies on the perpendicular bisector of $\overline{AB}$. Conversely, a point on the perpendicular bisector of $\overline{AB}$ is equidistant from A and B.

EXERCISES

A.

In exercises 1–4 decide whether the given information determines congruence by HL, HA, AAS, ASA, or none of these.

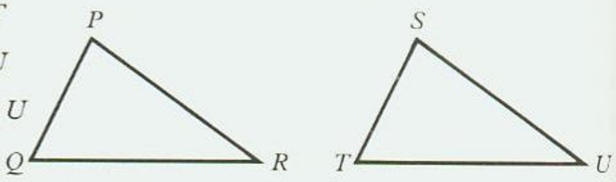
1. **Given:** $\overline{AB} \cong \overline{DE}$, $\overline{AC} \cong \overline{DF}$
2. **Given:** $\angle A \cong \angle D$, $\overline{BC} \cong \overline{EF}$.
3. **Given:** $\angle B \cong \angle E$, $\overline{AB} \cong \overline{DE}$
4. **Given:** $\overline{AC} \cong \overline{DF}$, $\angle A \cong \angle D$



(Exs. 1–4)

In exercises 5–9 decide whether the given conditions insure that the triangles are congruent. If they are not, provide a counterexample.

5. **Given:** $\overline{PQ} \cong \overline{ST}$, $\angle P \cong \angle S$, $\angle Q \cong \angle T$
6. **Given:** $\overline{PQ} \cong \overline{TU}$, $\overline{QR} \cong \overline{SU}$, $\angle Q \cong \angle U$
7. **Given:** $\angle P \cong \angle S$, $\angle Q \cong \angle T$, $\angle R \cong \angle U$
8. **Given:** $\angle Q \cong \angle T$, $\overline{PQ} \cong \overline{ST}$, $\overline{PR} \cong \overline{SU}$
9. **Given:** $\overline{PQ} \cong \overline{SU}$, $\overline{QR} \cong \overline{ST}$, $\overline{PR} \cong \overline{TU}$

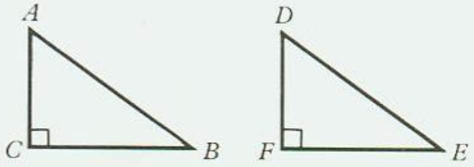


(Exs. 5–9)

B.

For exercises 10–11 assume that $\overline{AB} \cong \overline{DE}$ and $\overline{AC} \cong \overline{DF}$.

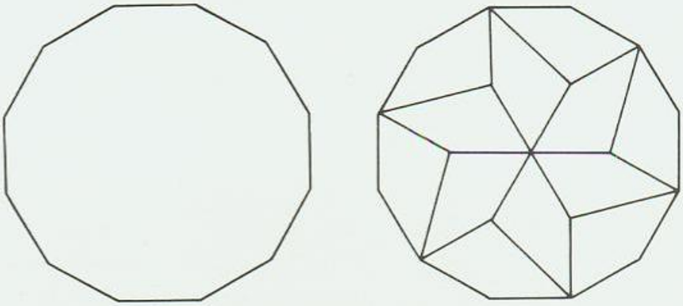
10. If $BC = 2x - 3$ and $EF = x + 5$, find the lengths BC and EF.
11. If $m\angle A = 4x - 5$ and $m\angle D = 2x + 25$, find $m\angle B$ and $m\angle E$.



(Exs. 10, 11)

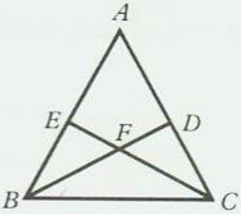
Activity

Trace and cut out all 13 pieces of these two smaller regular 12-gons and arrange them to form a large regular 12-gon.



12. **Given:** $\overline{BD} \cong \overline{CE}$
    $\overline{BD}$ and $\overline{CE}$ are altitudes.
    Prove: $\triangle ABC$ is isosceles.

13. **Given:** $\overline{BD}$ and $\overline{CE}$ are altitudes.
    $\angle ABC \cong \angle ACB$
    Prove: $\triangle BFC$ is isosceles.



(Exs. 12, 13)

C.

For exercises 14–16, $\overline{BE} \cong \overline{CD}$ and $\overline{BD}$ and $\overline{CE}$ are altitudes.

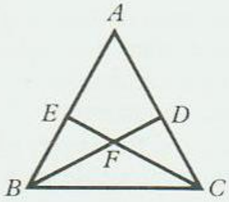
14. Prove that $\triangle EFB \cong \triangle DFC$.

15. Prove that $\triangle AED$ is isosceles.

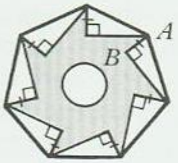
16. Prove that $\overleftrightarrow{ED} \parallel \overleftrightarrow{BC}$.

17. Prove Theorem 6–10.

18. A milling cutter with seven teeth is made by cutting seven right triangles out of a 7-sided regular polygon. If $\overline{AB}$ is cut the same length for each tooth, why are the sharp points of the cutter all the same size angle?

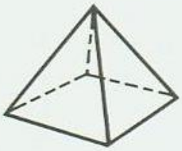


(Exs. 14–16)



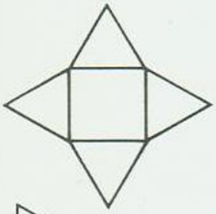
PROBLEM SOLVING

Pictured at the right is a square based pyramid.

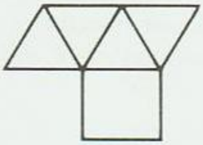


1. Which of these networks can be folded into the shaped pyramid?

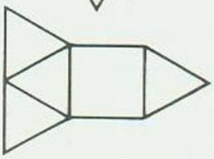
A



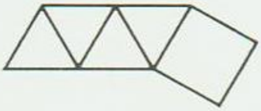
B



C



D



2. Make larger networks like each of the above. Check your guess by trying to make a model of each.

Important Ideas—Chapter 6

Terms

Scalene triangle (p. 198)
Acute triangle (p. 198)
Right triangle (p. 199)
Hypotenuse (p. 199)
Obtuse triangle (p. 199)
Equiangular triangle (p. 199)
Altitude (p. 199)

Theorems

6–1 If a triangle is isosceles, then its base angles are congruent.

6–2 If a triangle is equilateral, then it is equiangular.

6–3 If two angles of a triangle are congruent, then the sides opposite these angles are congruent.

6–4 The sum of the measures of the angles of a triangle is 180°.

6–5 The angles of an equilateral triangle each have a measure of 60°.

6–6 The measure of an exterior angle of a triangle is equal to the sum of measures of its two remote interior angles.

6–7 **AAS Theorem.** If two angles and a side opposite one angle in a triangle are congruent to two angles and the corresponding side of a second triangle, then the two triangles are congruent.

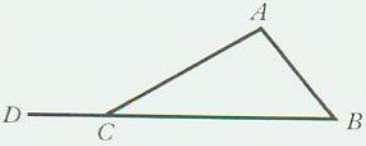
6–8 **HA Theorem.** If the hypotenuse and an acute angle of one right triangle are congruent to a hypotenuse and an acute angle of another right triangle, then the triangles are congruent.

6–9 **HL Theorem.** If the hypotenuse and a leg of one right triangle are congruent to the hypotenuse and a leg of a second right triangle, then the triangles are congruent.

6–10 If a point P is equidistant from a pair of points A and B, then P lies on the perpendicular bisector of $\overline{AB}$. Conversely, a point on the perpendicular bisector of $\overline{AB}$ is equidistant from A and B.

Chapter 6—Review

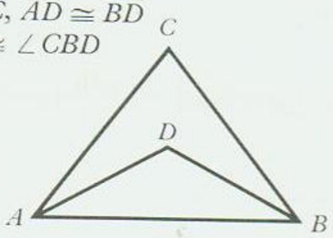
1. Indicate whether the following statements are true or false.
   - **a.** A scalene triangle can never be an acute triangle.
   - **b.** An isosceles triangle can also be a right triangle.
   - **c.** A right triangle has exactly one altitude.
   - **d.** If two angles of a triangle have measures of 60° and 90°, then the triangle is isosceles.



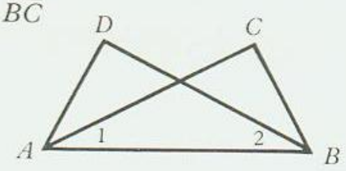
2. If $m\angle ACD = 120$ and $m\angle ABC = 50$, find $m\angle BAC$.

3. The measures of the two base angles of an isosceles triangle are represented by $x + 20$ and $3x - 40$. Find the measure of the vertex angle.

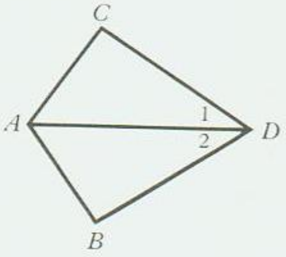
4. **Given:** $\overline{AC} \cong \overline{BC}$, $\overline{AD} \cong \overline{BD}$
Prove: $\angle CAD \cong \angle CBD$



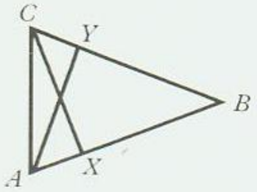
5. **Given:** $\angle C \cong \angle D$
$\angle 1 \cong \angle 2$
Prove: $\overline{AD} \cong \overline{BC}$



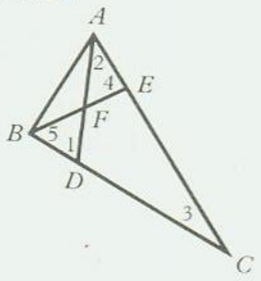
6. **Given:** $m\angle B = m\angle C = 90$
$\angle 1 \cong \angle 2$
Prove: $\overline{BD} \cong \overline{CD}$



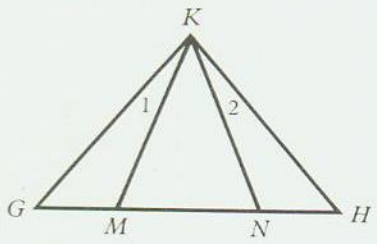
7. **Given:** $\overline{CX}$ is an altitude to $\overline{AB}$.
$\overline{AY}$ is an altitude to $\overline{CB}$.
$\overline{CX} \cong \overline{AY}$
Prove: $\triangle ABC$ is isosceles



8. **a.** $m\angle DFE = m\angle 5 + m\angle 1 = m\angle 2 + m\angle 4$. Why?
   b. If $m\angle 4 = 80$ and $m\angle 3 = 35$, find $m\angle 5$.

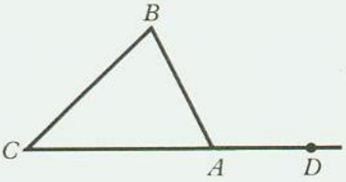


9. **Given:** $\triangle KMN$ is isosceles.
$\angle 1 \cong \angle 2$
Prove: $\triangle KHG$ is isosceles.

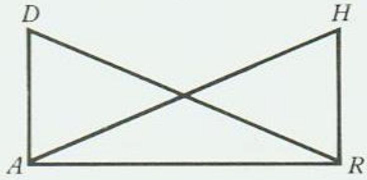


Chapter 6—Test

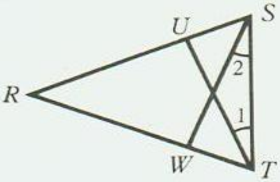
1. Indicate whether the following statements are true or false.
   - **a.** An equilateral triangle is equiangular.
   - **b.** An obtuse triangle can have a right angle.
   - **c.** If an acute angle of one right triangle is congruent to an acute angle of another right triangle, then the two triangles are congruent.
   - **d.** If two angles of a triangle have measures of 100° and 40°, then the triangle is isosceles.
2. If $\overline{AB} \cong \overline{BC}$ and $m\angle BAD = 116$, find $\angle B$.



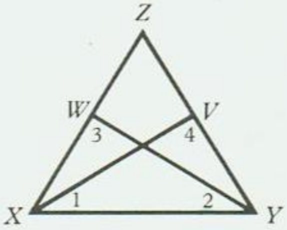
3. The measures of the angles of a triangle are represented by $4x$, $5x - 5$ and $x + 35$. Find the measures of each angle.
4. **Given:** $\overline{DA} \perp \overline{AR}$, $\overline{HR} \perp \overline{AR}$
   $\overline{DR} \cong \overline{HA}$
   Prove: $\overline{DA} \cong \overline{HR}$



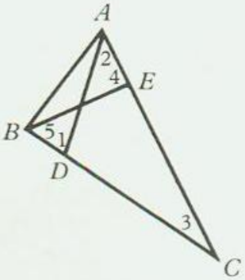
5. **Given:** $\overline{TU}$ is the altitude to $\overline{RS}$
   $\overline{SW}$ is the altitude to $\overline{RT}$
   $\angle 1 \cong \angle 2$
   Prove: $\triangle RST$ is isosceles



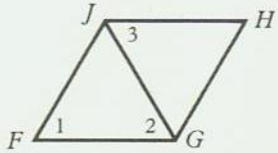
6. **Given:** $\angle 1 \cong \angle 2$
   $\angle 3 \cong \angle 4$
   Prove: $\overline{XV} \cong \overline{YW}$



7. Prove that the altitude to the base of an isosceles triangle is also the angle bisector of the vertex angle.
8. **a.** If $m\angle 2 = 20$ and $m\angle 3 = 35$, find $m\angle 1$.
   b. If $m\angle BEC = 100$ and $m\angle BAE = 65$, find $m\angle ABE$.



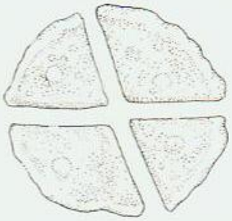
9. **Given:** $\overline{JH} \parallel \overline{FG}$
   $\angle 1 \cong \angle 3$
   $\overline{JH} \cong \overline{JF}$
   Prove: $\triangle JGH$ is isosceles.



Problem Solving Techniques

Make a Table-II

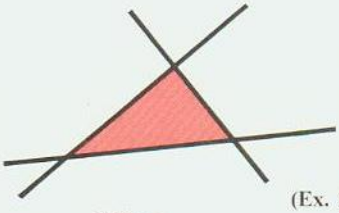
In a previous section on problem solving techniques we suggested making a table to help solve problems. Here are some more problems in which making a table can be useful.



2 cuts, 4 pieces
8 cuts, how many pieces?

PROBLEMS

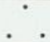
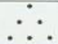
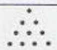
For each problem, make a table, observe a pattern, and solve the problem.



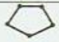
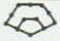
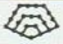
(Ex. 2)

3 lines
1 bounded region

1. What is the maximum number of pieces of pizza that can be obtained if the pizza is cut 8 times? (*Hint:* Construct a table. One column should be the number of cuts and the second column should be the maximum number of pieces.)
2. A region of the plane that is bounded on all sides by a line is called a *bounded region.* How many bounded regions are formed if 9 lines are drawn?
3. A triangular number (T) is one that can be represented geometrically as shown. T_1, T_2, T_3, and T_4 are illustrated on the right. Can you find T_5 and T_6? Can you predict what the tenth triangular number (T_{10}) is?

	configuration	number
T_1	·	1
T_2		3
T_3		6
T_4		10
T_5	?	?
T_6	?	?

4. A pentagonal number (P) is another number that can be represented geometrically as shown. Can you find P_5 and P_6? Can you predict what the tenth pentagonal number (P_{10}) will be?

	configuration	number
P_1	·	1
P_2		5
P_3		12
P_4		22
P_5	?	?
P_6	?	?

5. A heptagonal number (H) is represented geometrically by drawing heptagons (seven sides) in a manner similar to problems 3 and 4. Find H_3 and H_4. Can you predict what the tenth heptagonal number (H_{10}) will be?